Optimal Channel Assignment for Military MANET Using Integer Optimization and Constraint Programming

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***Abstract*—The military is developing, purchasing, and fielding mobile ad-hoc network (MANET) radios capable of connecting highly mobile units operating in rugged terrain over long distances. These radios offer tremendous new capabilities, including high data rates and automatic traffic relay, but have large electromagnetic spectrum requirements. We explore the challenges faced by a spectrum manager in allocating the minimum number of channels to support communications for military forces conducting tactical operations. In previous work, we identified the vast computational challenges of solving this problem when considering the effects of cumulative co-channel interference. We present a new method to solve this problem using heuristic, integer optimization, and constraint programming techniques. We apply our methods to realistic data sets from a large U.S. Marine Corps combat scenario, and provide detailed performance results. We also outline our plan for further developing this method to consider multiple time steps. To our knowledge, we are the first to describe an algorithm for solving a realistic, large-scale interference-aware minimum-order channel assignment problem to global or near-global optimality.**

I. Introduction

The United States military is currently developing, purchasing, and fielding mobile ad-hoc network (MANET) radios capable of connecting highly mobile units operating in rugged terrain over long distances. While these radios offer tremendous new capabilities, including high data rates and automatic traffic relay, they have large electromagnetic (EM) spectrum requirements. Meanwhile, the U.S. military will continue to operate in spectrum-constrained environments, both in the U.S. and abroad. Civilian, joint, and coalition communications transmissions will increasingly clutter the EM spectrum, and the Federal Communications Commission (FCC) is reassigning the military to new bands to share spectrum with the private sector [1], [2].

Efficient allocation of available EM *channels* is necessary to fully utilize new MANET radios [3], yet current allocation methods are inadequate. Indeed, in a major study the U.S. Marine Corps finds that with current allocation methods, Marine task forces will have insufficient spectrum to support the use of wideband MANET radios in major combat operations [4].

We consider the problem of a U.S. Marine Corps (USMC) *spectrum manager* who must determine an efficient spectrum allocation scheme to support multiple, mobile, independent MANETs operating on rough terrain. Many Marine Corps EM systems, including single-channel radios, radars, jammers, and the independent MANETs we consider, do not automatically coordinate channel assignments because of security concerns, additional complexity, and communications overhead (i.e., bandwidth and processing required for coordination). They receive centralized channel assignments from a spectrum manager, and then a human operator manually configures the radios. Using radio locations, performance specifications, and terrain elevation data, our spectrum manager attempts to minimize the number of channels required to support communications while being mindful of *co-channel interference* (unintentional electromagnetic transmissions between two or more radios assigned the same channel). We assume our spectrum manager has local access to a multi-core personal computer.

Spectrum managers currently use several software tools to support spectrum allocation, including the Systems Planning, Engineering, and Evaluation Device (SPEED) [5] and Spectrum XXI [6]. While these tools provide radio coverage analysis reports and a database to deconflict assignments, neither consider interference among a large number of mobile transmitters, nor do they provide a method for minimizing the number of required channels.

Metzger [7] is credited with first observing the possibility of using optimization techniques for solving the *channel assignment problem* (*CAP*). He relates the problem to the classic *graph-coloring problem*, which constrains any two adjacent nodes (i.e., radios) from being assigned the same color (i.e., channel). These pairwise constraints are used in the vast majority of research on the *interference-aware* CAP [8]–[11]. This basic form of the CAP is *NP-complete* [12], yet the more realistic *cumulative interference constraints* that must be considered for military MANETs make the problem far more computationally taxing.

In previous work, we describe in detail these computational challenges [13]. In the present work, we use heuristic and integer optimization and constraint programming techniques to develop a new method for solving this difficult and important problem. To our knowledge, we are the first to describe an algorithm for solving a realistic, large-scale interference-aware minimum-order channel assignment problem to global or near optimality.

In the next section, we provide our cumulative-interference

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CAP formulation and describe our realistic datasets generated from a large USMC combat scenario. In Section III, we briefly summarize the computational challenges of this problem, and then describe our solution methods and their performance. We conclude with an overview of future research.

II. Problem Formulation and Datasets

A. Cumulative-interference minimum-order channel assignment problem

We represent and solve our cumulative-interference minimum-order channel assignment problem as an integer optimization problem or *integer program* (*IP*), building on the formulations of [4], [9], [13], [14]. Let $r \in R$ (alias s) represent each MANET radio. Each radio is permanently assigned to a MANET *unit* $u \in U$, indicated by the set of *logical arcs* $(r, u) \in L$. A unit represents a tactical military organization, such as an infantry company or battalion headquarters. Let the set of *nodes* N (indexed by n) comprise both radios R and units U, i.e., $n \in N = R \cup U$.

Let a channel $c \in C$ be a contiguous range of EM frequencies, where C is the set of available *orthogonal* (i.e., non-interfering) channels. Each unit u and the radios assigned to it require a channel assignment. Let $X_n^c \in \{0, 1\}$ indicate whether node n (either a radio or a unit) is using channel c:

$$X_n^c = \begin{cases} 1, & \text{if node } n \text{ uses channel } c \\ 0, & \text{otherwise} \end{cases} \quad \forall n \in N, c \in C. \tag{1}$$

All radios in a unit use the same channel, so:

$$X_r^c = X_u^c \quad \forall c \in C, (r, u) \in L \tag{2}$$

and each unit u is assigned only one channel, so:

$$\sum_{c \in C} X_u^c = 1 \quad \forall u \in U. \tag{3}$$

Let $Y^c \in \{0, 1\}$ indicate whether channel c is being used:

$$Y^c = \begin{cases} 1, & \text{if channel } c \text{ is used} \\ 0, & \text{otherwise} \end{cases} \quad \forall c \in C \tag{4}$$

which is enforced via:

$$X_u^c \leq Y^c \quad \forall u \in U, c \in C. \tag{5}$$

Let $(r, s) \in W$ indicate the set of all wireless arcs between all radios $r, s \in R$. These arcs represent both intentional EM transmissions between radios assigned to the same unit, and unwanted interference from all other radios assigned to the same channel $c \in C$. Each unit $u \in U$ forms a separate MANET among its assigned radios using the available wireless arcs $(r, s) \in W : (r, u) \in L, (s, u) \in L$. In our scenarios, there are no connections between units; that is, disparate MANETs are not connected via a *backhaul network*.

We use a *signal-to-interference ratio* (*SIR*) model to calculate the strength of both co-channel interference and purposeful wireless transmissions between intra-unit radios. We consider only co-channel interference, as *adjacent-channel* and other *harmonic* interference are negligible due to orthogonal separation and adequate *white space* between channels. We calculate the *received signal strength* (*RSS*) along all wireless arcs $(r, s) \in W$ in watts using the standard link budget formula [15]. While SIR is not the only consideration in determining radio performance, it is generally the limiting factor in determining the ability to reuse a channel [16], [17], especially in the scenarios we consider where terrain and mobility greatly affect radio propagation [4], [18]. We use the Terrain Integrated Rough Earth Model (TIREM) of Alion Science & Technology Corporation [19] to calculate total path loss over terrain; other common methods include the Irregular Terrain Model (ITM) [20] and Hata-COST 231 [21].

The magnitude of co-channel interference along each arc $(r, s) \in W$ for each available channel $c \in C$ is indicated by $\textit{interference}_{rs}^c$. For each radio, we pre-calculate the maximum allowable interference $\textit{max\_interference}_s^c$ before a radio s is disconnected from the rest of its MANET (see [4] for details). Following [8], [10], [16], [22], [23], we consider the cumulative effect of all interference sources. That is, a radio may be unable to use a channel because the total sum of interference exceeds $\textit{max\_interference}_s^c$. Summing along all arcs on channel c yields:

$$\sum_{r:(r,s) \in W} \textit{interference}_{rs}^c X_r^c X_s^c \leq \textit{max\_interference}_s^c \quad \forall s \in R, c \in C. \tag{6}$$

To linearize these constraints, we introduce the variable $Z_{rs}^c \in \{0, 1\}$ where:

$$Z_{rs}^c = \begin{cases} 1, & \text{if } X_r^c = X_s^c = 1 \\ 0, & \text{otherwise} \end{cases} \quad \forall r, s \in R, c \in C \tag{7}$$

which is enforced via:

$$Z_{rs}^c \geq X_r^c + X_s^c - 1 \quad \forall r, s \in R, c \in C \tag{8}$$
$$Z_{rs}^c \leq X_r^c \quad \forall r, s \in R, c \in C \tag{9}$$
$$Z_{rs}^c \leq X_s^c \quad \forall r, s \in R, c \in C. \tag{10}$$

We thus obtain our cumulative co-channel interference constraints:

$$\sum_{r:(r,s) \in W} \textit{interference}_{rs}^c Z_{rs}^c \leq \textit{max\_interference}_s^c \quad \forall s \in R, c \in C. \tag{11}$$

The goal of minimum-order CAP (MO-CAP) is to minimize the total number of required channels, so its objective function is:

$$\min \sum_{c \in C} Y^c. \tag{12}$$

MO-CAP is pure 0-1 integer program, and comprises constraints (2)-(3), (5), (8)-(11) and objective function (12).

B. Realistic Datasets

To illustrate the utility of our technique, we use realistic datasets depicting particular time steps within high-fidelity simulations of Marine Corps combat operations. We use Systems Toolkit (STK) [24] to develop our scenarios, i.e., to position radios in time and space according to the scenario concept of operations, and then use Python and TIREM [19] to calculate radio propagation between all radios at each of

TABLE I. SIZE OF SCENARIOS BY NUMBER OF MARINES, UNITS, AND RADIOS, AND ASSOCIATED MO-CAP RELATIVE OPTIMALITY GAP AND APPROXIMATE SOLUTION TIMES USING "BRUTE FORCE" CPLEX TECHNIQUE.

Scenario	Marines	Units	Radios	Relative Optimality Gap	Solution Time
MEU	2000	6	131	0%	$<$ 2 sec
MEB	15,000	24	641	0%	24 hours
MEF	60,000	118	1887	77%	$>$ 60 hours

twenty time steps. We consider three tactical *Marine Air-Ground Task Force* (*MAGTF*) scenarios, each with different network topologies. The first scenario, based on Major Combat Operation 1 [25] involves a Marine Expeditionary Unit (MEU) conducting an amphibious assault on an island. The second scenario, based on combat operations in Helmand Province, Afghanistan circa January 2010, is a Marine Expeditionary Brigade (MEB) conducting irregular warfare (IW) operations in a desert environment. Our final scenario, based on Integrated Security Construct B [26], is a Marine Expeditionary Force (MEF) conducting a major amphibious assault. A summary of these scenarios and their associated number of Marines, units, and radios is displayed in Table I. Each unit requires a channel assignment to support its individual MANET, and a single MANET may support up to 30 radios. We find the MEF scenario presents the most interesting computational challenge, so we generate datasets at 20 different time steps within the scenario in order to explore it more thoroughly. Each time step is a separate dataset (each with 118 units comprising 1887 total radios), depicting the relative locations of units at that moment in time according to the *concept of operations* of the scenario. See [4] for further details on our scenarios.

III. SOLUTION METHODS AND COMPUTATIONAL RESULTS

As we describe in previous work [13], the cumulative interference MO-CAP presents several computational challenges. In the following sub-sections we summarize these challenges, and describe in some detail the methods we develop to solve the problem to global or near optimality. Unless otherwise noted, all results are obtained using a Dell Mobile Precision 6800 laptop with 32 GB of RAM and an Intel Core i7-4940MX processor running at 3.1 GHz. We use IBM ILOG CPLEX version 12.6.2 and Python 2.7.

A. Integer Optimization

Integer optimization techniques are the most common method of exactly solving the MO-CAP [9]. Generally, these methods navigate the *solution tree* by selecting variables to fix, solving the associated linear programming *relaxation*, adding additional constraints, and applying preprocessing and heuristics in order to tighten the upper and/or lower bounds and *fathom* (i.e., cut off) suboptimal portions of the tree [27].

Our realistic datasets present several computational challenges to the basic integer programming approach. First, LP solvers are sensitive to vast differences in input parameters [28], such as the values $interference^c_{rs}$ and $max\_interference^c_s$ which in our simulations vary by 24 orders of magnitude,

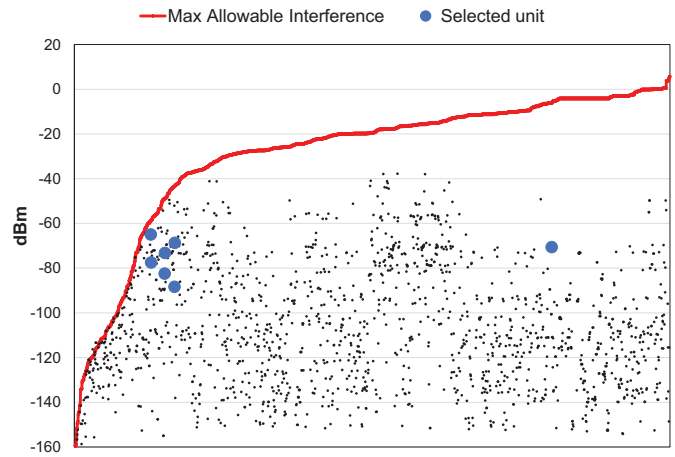

Fig. 1. Depiction of the received interference (dots) and interference threshold (red line) for each of the 1887 radios in the MEF scenario (time step one), after being solved by CPLEX with only pairwise constraints. Nine radio fall above the line (in the lower-left corner), indicating constraint violations. Large blue dots represent seven radios assigned to one particular unit.

and are generally quite small. Specifically, a solver may have difficulty in choosing an appropriate variable to fix and correctly rounding fractional values when the associated interference value may vary by such a large amount [29]. Another computational problem (also identified by [10]) is that of *symmetry*, which occurs when channel assignments may be changed without changing the objective value [30]. The very *near* symmetry that we observe in our datasets (as opposed to *exact* symmetry) is especially difficult for solvers to detect and mitigate [31], [32].

Perhaps the most difficult computational challenge of the MO-CAP are the cumulative interference constraints (11). LP solvers often leverage the *sparse* nature of a problem by considering only subsets of variables at a time, but cumulative interference constraints may create relatively dense input matrices. This makes preprocessing less productive, limits the ability of an LP solver to decompose the problem, and makes merely checking the satisfaction of a constraint more computationally taxing [33]. For these reason, the vast majority of exact optimization work on the MO-CAP considers only pairwise interference constraints [8]–[11].

We find in our scenarios that considering only pairwise interference constraints will cause at least a few radios to be disconnected from their respective MANETs. "Repairing" these disconnections, i.e., ensuring all radios in each unit are connected, is what makes this problem particularly challenging. Figure 1 provides a visualization of the received interference and interference constraints for the first time step of the MEF scenario (solved using CPLEX and considering only pairwise constraints). The received interference at each radio is indicated as a black dot, where the vertical axis indicates signal strength in dBm, and the horizontal axis displays the rank-ordered list of radios by interference (i.e., the radio receiving the least interference is on the extreme left). The red line (actually, collection of points) immediately above each radio indicates the $max\_interference^c_s$ threshold; a point above this line indicates a radio receives too much interference. Though difficult to see in this figure, there are

nine radios that receive excessive interference and are thus unable to communicate.

One can imagine trying to "push" these points under the line in Figure 1 by reassigning channels. The empty space under the red line in the upper-right corner seems to indicate there is slack in the constraints, i.e., that reassigning the violated radios should be easy, given that some radios receive interference far below their respective thresholds. In practice, this is extremely challenging, in part because radios within a particular unit are often spread across this diagram, i.e., the radios receive greatly different interference. To illustrate, the highlighted blue dots indicate radios from a single unit. While one radio has considerable slack, several radios are very close to their respective thresholds, and thus cannot easily be reassigned to a different channel with other radios.

Despite these challenges, we find that even a simple "brute force" integer optimization method (i.e., using CPLEX to solve the full problem as-is, without providing any initial solution or conducting preprocessing) is sufficient to solve the smaller two scenarios to optimality (see Table I). However, this approach fails to obtain useful answers to the MEF scenario, even after 60 hours of computation on a cluster of 14 high-performance desktop computers. For the remainder of this paper, we consider only the MEF scenario (at 20 time steps, each solved separately) since it presents a difficult computational challenge.

B. Heuristic

Heuristics are often used with the MO-CAP due to the described computational difficulties of exactly solving the problem [9], [23]. Heuristics that consider cumulative interference include *neighborhood search* [10], [34], *simulated annealing* [35], [36], *tabu search* [35], [37]–[39], *ant colony optimization* [40], *greedy heuristics* [4], [14], [41]–[44], and a combination of greedy and exact methods [10], [45].

In order to find an initial feasible solution, we create a simple greedy heuristic that iteratively "packs" units onto channels until the channel is "full," and then starts with the next channel (see [14] for pseudocode on our heuristic). The "sol'n" column under "Heuristic" in Table II displays the results of our heuristic at each of the 20 time steps (each solved separately) within the MEF scenario. In no case does the heuristic find the optimal solution, which isn't surprising given that our constructive heuristic stops after finding a feasible solution. We use the heuristic result as an initial feasible solution to the "brute force" CPLEX method. However, the solver fails to improve upon the initial solution, even when using a cluster of 14 high-performance desktops over a time period of two weeks.

While our constructive heuristic guarantees a feasible solution (as long as the number of available channels is at least as large as the number of units, i.e., $|C| \geq |U|$), it provides no *certificate of optimality*, i.e., the distance to the global optimum is unknown. We feel these bounds are important for understanding the goodness of a particular solution, especially since spectrum is so scarce. In the next section we develop an alternative formulation that both provides bounds on optimality and provides much better feasible solutions.

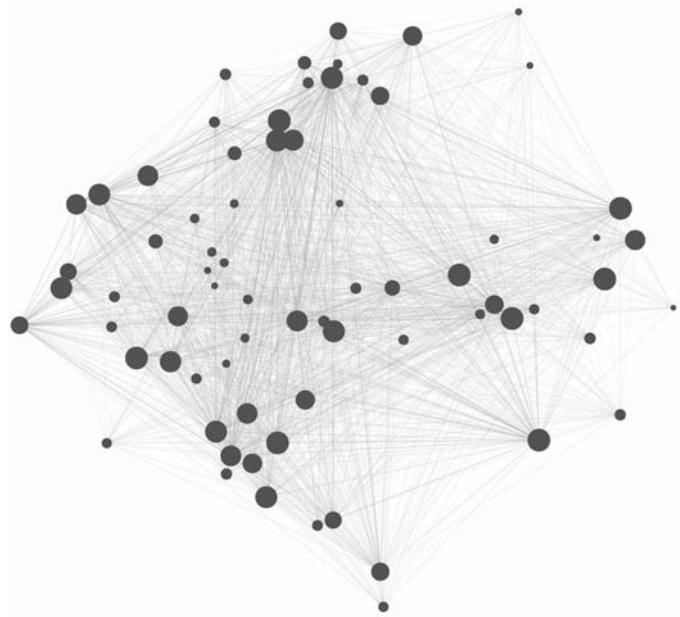

Fig. 2. Depiction of pairwise constraints in MEF time step 1, where an arc indicates that the two associated units cannot be assigned the same channel at the same time, and the size of each node is relative to the degree of the node.

C. Preprocessing and Lazy Constraints

We develop a method of preprocessing the input data in order to reduce the numerical issues associated with the $\mathit{interference}^c_{rs}$ and $\mathit{max\_interference}^c_s$ values. Given a dataset, we preprocess the cumulative interference constraints (11) to create simplified and more computationally tractable *packing constraints* (see, e.g., [46]). For example, suppose two specific nodes r and s (not assigned to the same unit) are not both allowed to be assigned to channel c because to do so would violate the associated interference constraint. This may be represented as:

$$X^c_r + X^c_s \leq 1. \tag{13}$$

We use Python and the `mpmath` library [47], which allows the use of arbitrary-precision floating point mathematics, to identify unacceptable combinations of radios and handle the extremely small interference values present in our data sets. Figure 2 shows the pairwise interference constraints for time step 1, where an arc between nodes indicates the two associated units cannot be assigned the same channel. We find that these *interference graphs* are fairly dense. On average across the time steps, there are 3653 pairs present, 53% of the total $|U||U-1|/2 = 6903$ possible. The "Pairwise Const." column of Table II displays the number of pairwise constraints in each time step.

To generalize for larger n-tuples of units above pairs (triplets, quadruplets, etc.), let $S \subset R$ be a subset of radios (none assigned to the same unit) that cannot all be assigned to the same channel c. We can represent such a restriction of assignments as:

$$\sum_{r \in S} X^c_r \leq |S| - 1. \tag{14}$$

TABLE II. SUMMARY OF COMPUTATIONAL RESULTS FOR THE HEURISTIC, INTEGER OPTIMIZATION, AND CP TECHNIQUES ON EACH OF TWENTY TIME STEPS (EACH SOLVED SEPARATELY) FROM THE MEF SCENARIO, USING A DELL MOBILE PRECISION 6800 LAPTOP WITH 32 GB OF RAM AND A 3.1 GHZ INTEL CORE I7-4940MX PROCESSOR. GAP INDICATES THE RELATIVE DISTANCE FROM THE GIVEN SOLUTION TO THE TRUE LOWER BOUND (IF KNOWN); **BOLD** INDICATES THE DISCOVERY OF A PROVEN LOWER BOUND.

	Heuristic			Integer Optimization						Constraint Programming			Best Known	
Time step	**Sol'n**	**Gap**	**Time (s)**	**Sol'n**	**Gap**	**Time (s)**	**% Improv.**	**Pairwise Const.**	**Lazy Const.**	**Infeas.**	**Time (s)**	**New LB?**	**Lower Bound**	**Gap**
1	51	9.80%	292.66	46	0.00%	1732.42	9.80%	4407	106	**45**	0.03	Yes	46	0.00%
2	48	22.92%	350.75	**37**	0.00%	1712.63	22.92%	3892	25	36	0.02		37	0.00%
3	46	$\geq$ 26.09%	340.53	36	$\geq$ 5.56%	4725.53	21.74%	3945	122	33	0.02		34 - 36	5.56%
4	47	$\geq$ 29.79%	379.69	34	$\geq$ 2.94%	2448.08	27.66%	3823	68	32	0.02		33 - 34	2.94%
5	43	23.26%	339.25	**33**	0.00%	1011.58	23.26%	3762	5	32	0.02		33	0.00%
6	51	$\geq$ 31.37%	333.76	36	$\geq$ 2.78%	1815.02	29.41%	3904	58	34	0.02		35 - 36	2.78%
7	49	24.49%	358.94	**37**	0.00%	1536.13	24.49%	3884	32	36	0.02		37	0.00%
8	42	$\geq$ 28.57%	342.33	31	$\geq$ 3.23%	9757.02	26.19%	3538	85	29	0.02		30 -31	3.23%
9	43	25.58%	371.08	**32**	0.00%	1963.9	25.58%	3398	30	31	0.02		32	0.00%
10	49	30.61%	348.66	**34**	0.00%	994.17	30.61%	3541	5	33	0.02		34	0.00%
11	45	26.67%	354.01	33	0.00%	4228.31	26.67%	3449	119	**32**	0.03	Yes	33	0.00%
12	43	$\geq$ 18.60%	298.93	36	$\geq$ 2.78%	2948.95	16.28%	3367	85	34	0.02		35 - 36	2.78%
13	43	25.58%	311.49	32	0.00%	2991.59	25.58%	3550	117	**31**	0.02	Yes	32	0.00%
14	43	$\geq$ 30.23%	321.6	31	$\geq$ 3.23%	3147.21	27.91%	3301	181	29	0.02		30 -31	3.23%
15	49	22.45%	409.84	38	0.00%	2218.54	22.45%	3666	55	**37**	0.01	Yes	38	0.00%
16	47	$\geq$ 25.53%	358.38	42	$\geq$ 16.67%	2529.19	10.64%	3749	52	34	0.02		35 - 42	16.67%
17	49	24.49%	328.1	37	0.00%	2689.3	24.49%	3721	81	**36**	0.03	Yes	37	0.00%
18	40	22.50%	324.81	**31**	0.00%	639.63	22.50%	3214	10	30	0.01		31	0.00%
19	40	25.00%	297.56	30	0.00%	2033.09	25.00%	3282	117	**29**	0.01	Yes	30	0.00%
20	49	24.49%	345.82	**37**	0.00%	1625.01	24.49%	3660	49	36	0.02		37	0.00%

Preprocessing all such unacceptable combinations and adding them as constraints may replace the cumulative co-channel interference constraints (11). However, identifying all unacceptable combinations would be very computationally costly (as they grow exponentially in number with both the number of units and available channels) and unnecessary, as many combinations will be redundant and/or represent negligible levels of co-channel interference.

Instead, we dynamically add these constraints to the formulation only as needed via *lazy constraints*, which are constraints which are feasible in the full version of the problem, but are only checked by the solver on an as-needed basis [29]. First, we calculate all pairwise interference constraints and add them to an initial version of the problem. We send this problem to CPLEX, and indicate to the solver that we wish to initiate lazy constraints *callbacks*. Upon finding a solution that is feasible (with the current constraints), the solver runs our lazy constraint callback code. The code checks the feasibility of the current CPLEX solution in the full problem; this can be calculated in polynomial time, specifically $O\left(|R|^2|C|\right)$). If infeasibility exists, we add packing constraints (14) to prevent the same units from being assigned the same channel again. CPLEX then continues the search process with these new constraints added into the formulation. The process repeats until optimality is achieved or a time limit is reached. The "Lazy Const." column in Table II displays the number of lazy constraints added for each time step in the MEF scenario.

This method avoids the problem of very small numbers in CPLEX, as we can process the constraints in Python, and then add the much-simplified packing constraints (14) in CPLEX. Also, since the solver is no longer required to calculate cumulative interference, the CPLEX problem no longer requires the index $r \in R$. We can remove this index and greatly reduce the number of decision variables in the CPLEX problem.

We find this method provides very good performance. Table II shows the comparative results, where each row indicates a separate time step within the MEF scenario. Our integer optimization method improves upon the heuristic solution with an average reduction of 23.4% required channels. For seven of the time steps, CPLEX is able to find a provably optimal solution (i.e., an optimality gap of zero), indicated in bold. However, this does come at a cost: the CPLEX method takes on average nearly seven times longer than the heuristic, although average runtimes of about 44 minutes (or about 38 minutes if time step 8 is dismissed as an outlier) are not unreasonable, and could certainly be improved with a more efficient programming language (e.g., C++) and additional processing resources.

In 13 cases, CPLEX fails to find an optimal solution, though it does provide a measure of the relative optimality gap. While this provides a useful lower bound, we wish to know with certainty if the solution may be improved. We turn to another mathematical technique to provide this certainty.

D. Constraint Programming

Constraint satisfaction or *constraint programming* (*CP*) techniques determine if there exists a consistent assignment of variables that satisfies a system of logical constraints. Whereas integer optimization is focused on maximizing or minimizing an objective function, CP focuses on satisfying constraints. CP is useful for handling logical constraints that would otherwise be difficult to formulate using traditional integer optimization techniques, and have been found to be a very useful complement to integer optimization by quickly ruling out infeasible solutions, such as within a *Benders decomposition framework* (see, e.g., [48]–[50]).

The MO-CAP can be expressed as a CP problem, as first suggested by [51]. CP is used to solve the MO-CAP (usually in conjunction with other techniques) by [8], [10], [36], [39], [52]. We use CP techniques to attempt to establish true lower bounds for those time steps that CPLEX is unable to provide a zero optimality gap. We use the Optimization Programming Language (OPL) [53] to formulate the problem using integer variables, where each variable $W_u \in C$ indicates the channel that unit u is assigned. We add all pairwise constraints to the problem, and solve the problem using IBM CP Optimizer [54]. We decrease the number of available channels (i.e., the domain of each variable) until the solver discovers that the problem is infeasible. If such a *relaxation* of the original problem is infeasible with the given number of channels, then we have established that the original problem (with all constraints) is also infeasible. This indicates that at least $|C| + 1$ channel are required, creating a lower bound. If the lower bound equals the upper bound, we have obtained an optimal solution.

The results are displayed in Table II, where "Infeas." indicates the number of channels at which the solver indicates the problem is infeasible, i.e., at least one more channel is required. On 13 time steps, CP Optimizer is able to establish an exact lower bound, and is able to do so in fractions of a second. For six of these time steps, CP Optimizer tightens the lower bounds found by CPLEX, i.e., it reduces the optimality gap to zero. On seven other time steps, we are unable to tighten the lower bound, even after considerable runtimes (over 12 hours) and the addition of *symmetry-breaking* constraints. A high degree of symmetry exists in this problem in that many different solutions (i.e., assignments of channels to units) have the same objective value. For example, the channel assignments of any two units may be swapped without changing the objective. Symmetry-breaking constraints reduce or eliminate this possibility, and may (though without certainty) provide better CP performance [55]. Following [56], we add such constraints to our MO-CAP CP formulation, but in each case, we fail to find a tighter lower bound. We also try adding all triplet constraints, as well as adding constraints iteratively, to no avail. We find in general that this CP approach is very efficient at finding infeasibilities but may struggle to find a feasible solution close to the lower bound.

IV. Conclusions and Future Work

We present a new method that integrates heuristic and integer optimization and constraint programming techniques to solve the cumulative-interference MO-CAP to global or near optimality relatively quickly. This technique could be useful to military spectrum managers in making spectrum allocation decisions in support of MANET operations.

Our immediate next step in pursuit of global optima at each time step is to find *irreducible infeasible sets* [52], in an effort to find the minimal number of channels that may be assigned to smaller parts of the problem (perhaps by geographic region). In future work, we plan to explore methods of solving this problem over multiple time steps. In fact, the present work can viewed as a part of a decomposition of a larger problem, where we first solve individual time steps to minimize the number of required channels, and then "connect" the time steps in a manner to minimize the number of required channel changes over time (recall our radios require manual configuration). Our initial computational experiments reveal that solving both problems simultaneously is intractable, even with our new optimization approach and when solving for only two time steps concurrently.

We are also interested in further improvements to our method, including more effective methods of employing constraint programming (specifically, adding constraints dynamically instead of resolving the problem). We are intrigued by the related *minimum-interference* problem, which considers the challenge of a spectrum manager who has only a certain number of channels available and must assign them in order to minimize interference. Aardal et al. [9] note that in general this is a much harder problem, as indicated by the relative dearth of research.

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